Series EF1GH/C

SET~4

प्रश्न–पत्र कोड
Q.P. Code 465

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

व्यावहारिक गणित

APPLIED MATHEMATICS

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निर्धारित समय : 3 घण्टे — अधिकतम अंक : 80
Time allowed : 3 hours — Maximum Marks : 80

नोट / NOTE :

(i) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।
Please check that this question paper contains **23** printed pages.

(ii) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।
Please check that this question paper contains **38** questions.

(iv) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
Please write down the serial number of the question in the answer-book before attempting it.

(v) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में **38** प्रश्न हैं । **सभी** प्रश्न **अनिवार्य** हैं ।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ** ।*

(iii) ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **एक-एक** अंक के प्रश्न हैं ।*

(iv) ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **दो-दो** अंकों के प्रश्न हैं ।*

(v) ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **तीन-तीन** अंकों के प्रश्न हैं ।*

(vi) ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **पाँच-पाँच** अंकों के प्रश्न हैं ।*

(vii) ***खण्ड ङ** में प्रश्न संख्या **36** से **38** प्रकरण अध्ययन आधारित **चार-चार** अंकों के प्रश्न हैं ।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 2 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है ।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है ।

1. यदि $100 \equiv x \pmod 7$ है, तो x का न्यूनतम धनात्मक मान है :

(a) 6 (b) 4

(c) 3 (d) 2

2. 1 किलोमीटर की एक दौड़ में A, B को 50 m अथवा 10 सेकंड से पीछे छोड़ देता है । A द्वारा दौड़ पूरी करने में लिया गया कुल समय है :

(a) 90 सेकंड (b) 120 सेकंड

(c) 190 सेकंड (d) 200 सेकंड

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $100 \equiv x \pmod 7$, then the least positive value of x is :

(a) 6 (b) 4

(c) 3 (d) 2

2. In a kilometre race, A beats B by 50 metres or 10 seconds. The time taken by A to complete the race is :

(a) 90 seconds (b) 120 seconds

(c) 190 seconds (d) 200 seconds

3. एक व्यक्ति एक नाव को धारा के अनुकूल 32 km ले जाने में अथवा धारा के प्रतिकूल 14 km ले जाने में, प्रत्येक दशा में 6 घंटे लेता है, तो धारा की चाल है :

(a) 2 km/h (b) 1·5 km/h

(c) 2·5 km/h (d) 2·25 km/h

4. यदि $-3x + 17 < -13$ है, तो :

(a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$

(c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10]$

5. A तथा B दो ऐसे आव्यूह हैं कि AB = A और BA = B है, तो B^2 बराबर है :

(a) B (b) A

(c) I (d) O

6. यदि $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ एक सममित आव्यूह है, तो :

(a) x = 0, y = 5 (b) x = 5, y = 0

(c) x = y (d) x + y = 0

7. x का वह मान जिसके लिए बिंदु (2, –1), (–3, 4) तथा (x, 5) संरेख हैं, है

(a) – 4 (b) 4

(c) 2 (d) – 2

3. If a man rows 32 km downstream and 14 km upstream in 6 hours each, then the speed of the stream is :

(a) 2 km/h (b) 1·5 km/h

(c) 2·5 km/h (d) 2·25 km/h

4. If $-3x + 17 < -13$, then :

(a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$

(c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10]$

5. If A and B are two matrices such that AB = A and BA = B, then B^2 is equal to :

(a) B (b) A

(c) I (d) O

6. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ is a symmetric matrix, then :

(a) $x = 0, y = 5$ (b) $x = 5, y = 0$

(c) $x = y$ (d) $x + y = 0$

7. The value of x for which the points $(2, -1)$, $(-3, 4)$ and $(x, 5)$ are collinear, is :

(a) -4 (b) 4

(c) 2 (d) -2

8. यदि $x + y = 8$ है, तो xy का अधिकतम मान है :

(a) 12 (b) 16

(c) 20 (d) 24

9. फलन $f(x) = x^x$, $x > 0$, जिस अंतराल में ह्रासमान है, वह है :

(a) $(-\infty, e)$ (b) $(0, e)$

(c) $\left(0, \frac{1}{e}\right)$ (d) $\left[\frac{1}{e}, \infty\right)$

10. $\int \frac{1}{x + x\log x}\, dx$ बराबर है :

(a) $1 + \log x + C$ (b) $x + \log x + C$

(c) $x \log (1 + \log x) + C$ (d) $\log (1 + \log x) + C$

11. यदि आपूर्ति फलन $p = 4 + x$ है, तो 12 इकाइयों की बिक्री पर उत्पादक का अधिशेष है :

(a) 72 (b) 64

(c) 76 (d) 46

12. वक्र कुल $y = Ae^{3x} + Be^{-3x}$ के संगत बने अवकल समीकरण की कोटि है :

(a) 1 (b) 2

(c) 3 (d) 4

8. If $x + y = 8$, then the maximum value of xy is :

(a) 12 (b) 16

(c) 20 (d) 24

9. The function $f(x) = x^x$, $x > 0$ is decreasing in the interval :

(a) $(-\infty, e)$ (b) $(0, e)$

(c) $\left(0, \frac{1}{e}\right)$ (d) $\left[\frac{1}{e}, \infty\right)$

10. $\int \frac{1}{x + x\log x} dx$ is equal to :

(a) $1 + \log x + C$ (b) $x + \log x + C$

(c) $x \log (1 + \log x) + C$ (d) $\log (1 + \log x) + C$

11. If the supply function is $p = 4 + x$, then the producer's surplus when 12 units are sold, is :

(a) 72 (b) 64

(c) 76 (d) 46

12. The order of the differential equation corresponding to family of curves $y = Ae^{3x} + Be^{-3x}$ is :

(a) 1 (b) 2

(c) 3 (d) 4

13. यदि प्वासों बंटन का माध्य 'm' है, तो $P(r = 0)$ है :

(a) e^{-m} (b) e^{m}

(c) e (d) m^{-e}

14. प्रसामान्य बंटन जिसके प्रति सममित है, वह है :

(a) प्रसरण (b) मानक विचलन

(c) माध्य (d) सह-प्रसरण

15. एक विद्यार्थी के t-test के लिए, परीक्षा आँकड़ा t निम्न द्वारा प्रदत्त है :

(a) $t = \bar{x} - \mu$ (b) $t = \frac{\bar{x}}{s}$

(c) $t = \frac{\bar{x} - \mu}{s}$ (d) $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$

16. दिए गए मानों 15, 23, 28, 36, 41, 46 के लिए 3-वर्षीय मूविंग एवरेज हैं :

(a) 24, 29, 35, 41 (b) 22, 28, 35, 41

(c) 22, 29, 35, 41 (d) 24, 28, 35, 41

17. अंकित ब्याज दर 8% वार्षिक, जो प्रति तिमाही संयोजित होती है, जिस प्रभावी ब्याज दर के तुल्य है, वह है : $[(1{\cdot}02)^4 = 1{\cdot}0824$ लीजिए]

(a) 8·16% (b) 7·95%

(c) 8·24% (d) 8·5%

18. व्यवरोधों $x \leq 3, y \leq 2, x \geq 0, y \geq 0$ के अंतर्गत फलन $z = 7x + 5y$, का अधिकतम मान है :

(a) 21 (b) 10

(c) 31 (d) 37

13. If 'm' is the mean of Poisson distribution, then P(r = 0) is given by :

(a) e^{-m} (b) e^{m}

(c) e (d) m^{-e}

14. Normal distribution is symmetric about :

(a) variance (b) standard deviation

(c) mean (d) covariance

15. For a student's t-test, the test statistic t is given by :

(a) $t = \bar{x} - \mu$ (b) $t = \frac{\bar{x}}{s}$

(c) $t = \frac{\bar{x} - \mu}{s}$ (d) $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$

16. For the given values 15, 23, 28, 36, 41, 46, the 3-yearly moving averages are :

(a) 24, 29, 35, 41 (b) 22, 28, 35, 41

(c) 22, 29, 35, 41 (d) 24, 28, 35, 41

17. The effective rate of return which is equivalent to nominal rate of 8% p.a. compounded quarterly is : [Given $(1{\cdot}02)^4 = 1{\cdot}0824$]

(a) 8·16% (b) 7·95%

(c) 8·24% (d) 8·5%

18. The maximum value of the function $z = 7x + 5y$, subject to constraints $x \leq 3, y \leq 2, x \geq 0, y \geq 0$, is :

(a) 21 (b) 10

(c) 31 (d) 37

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं और प्रत्येक प्रश्न का 1 अंक है । दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है । इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (a), (b), (c) और (d) में से चुनकर दीजिए ।*

(a) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है ।

(b) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है ।

(c) अभिकथन (A) सही है परन्तु तर्क (R) ग़लत है ।

(d) अभिकथन (A) ग़लत है परन्तु तर्क (R) सही है ।

19. *अभिकथन (A) :* वक्र $y = 2x^2 - 5x$ के अभिलंब की $x = -1$ पर प्रवणता -1 है।

तर्क (R) : वक्र $y = f(x)$ के अभिलंब की बिंदु (α, β) पर प्रवणता $(x - \alpha) + \left(\frac{dy}{dx}\right)_{(\alpha, \beta)} \cdot (y - \beta) = 0$ है।

20. *अभिकथन (A) :* यदि A कोटि 3 का एक वर्ग आव्यूह है और $|\text{adj } A| = 144$ है, तो $|A|$ का मान ± 12 है ।

तर्क (R) : यदि A एक व्युत्क्रमणीय आव्यूह है जिसकी कोटि n है, तो $|\text{adj } A| = |A|^{n-1}$.

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं ।

21. (क) असमिका $\frac{3}{5}x - \frac{2x-1}{3} > 1, x \in W$ का हल ज्ञात कीजिए।

अथवा

(ख) असमिका $-\frac{2}{3} < -\frac{x}{3} + 1 \leq \frac{2}{3}, x \in R$ का हल ज्ञात कीजिए।

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19. *Assertion (A) :* The slope of the normal to the curve $y = 2x^2 - 5x$ at $x = -1$ is -1.

Reason (R) : The slope of the normal to the curve $y = f(x)$ at point (α, β) is given by $(x - \alpha) + \left(\frac{dy}{dx}\right)_{(\alpha, \beta)} \cdot (y - \beta) = 0$.

20. *Assertion (A) :* If A is a square matrix of order 3 such that $|\text{adj } A| = 144$, then the value of $|A|$ is ± 12.

Reason (R) : If A is an invertible matrix of order n, then $|\text{adj } A| = |A|^{n-1}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Solve the inequality :

$$\frac{3}{5}x - \frac{2x-1}{3} > 1, x \in W$$

OR

(b) Solve the inequality :

$$-\frac{2}{3} < -\frac{x}{3} + 1 \leq \frac{2}{3}, x \in R$$

22. आव्यूह $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ को एक सममित तथा एक विषम-सममित आव्यूह के योग के रूप में लिखिए।

23. (क) यह दिया है कि एक कंपनी द्वारा बनाए गए पेंचों में से 2% खराब होते हैं। प्वासों बंटन के प्रयोग से प्रायिकता ज्ञात कीजिए कि 100 पेंचों के एक पैकिट में कोई भी खराब पेंच न हो।

(दिया है $e^{-2} = 0{\cdot}14$)

अथवा

(ख) यदि किसी प्वासों चर X का मानक विचलन $\sqrt{3}$ है, तो P (X > 0) ज्ञात कीजिए।

[$e^{-3} = 0{\cdot}05$ लीजिए]

24. हर छ: मास के अंत में ₹ 1000 के क्रमागत भुगतान, जो उसी प्रकार चलते रहें, का वर्तमान मूल्य ज्ञात कीजिए जबकि पूँजी पर 8% वार्षिक दर से प्रति छ:माही ब्याज लगता है।

25. एक मशीन, जो दो वर्ष पहले खरीदी गयी थी, का प्रतिवर्ष 10% वार्षिक दर से अवमूल्यन होता है। यदि इसका वर्तमान मूल्य ₹ 97,200 है, तो :

(i) इसका 3 वर्ष बाद का मूल्य ज्ञात कीजिए;

(ii) इस का वह मूल्य, जब यह खरीदी गई थी, ज्ञात कीजिए।

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. निम्नलिखित को एक रैखिक प्रोग्रामन समस्या के रूप में बदलें:

एक छोटी फर्म सोने की अंगूठी तथा चेन बनाती है। एक दिन में बनने वाली अंगूठियों तथा चेनों की अधिकतम संख्या 24 है। एक अंगूठी बनाने में 1 घंटा तथा एक चेन बनाने में 30 मिनट लगते हैं। एक दिन में अधिकतम 16 घंटे उपलब्ध हैं। यदि एक अंगूठी पर ₹ 300 तथा एक चेन पर ₹ 190 का लाभ होता है, तो अधिकतम लाभ के लिए प्रतिदिन में बनाई जाने वाले अंगूठियों तथा चेनों की संख्या ज्ञात कीजिए।

22. Write the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.

23. (a) It is given that 2% of screws manufactured by a company are defective. Using Poisson distribution, find the probability that a packet of 100 screws contains no defective screw.

(Given : $e^{-2} = 0{\cdot}14$)

OR

(b) If the standard deviation of a Poisson variable X is $\sqrt{3}$, then find P (X > 0). [Use : $e^{-3} = 0{\cdot}05$]

24. Find the present value of a sequence of payments of ₹ 1000 made at the end of every 6 months and continuing forever, if money is worth 8% per annum compounded semi-annually.

25. The value of a machine purchased two years ago, depreciates at the annual rate of 10%. If its present value is ₹ 97,200, find.

(i) its value after 3 years;

(ii) its value when it was purchased.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Formulate the following problem as an LPP :

A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit.

27. एक व्यक्ति एक नाव को स्थिर जल में 5 किमी प्रति घंटे की चाल से चला सकता है। उसे धारा के प्रतिकूल जाने में धारा के अनुकूल जाने से तीन गुना समय लगता है। धारा की चाल ज्ञात कीजिए।

28. एक बर्तन में 50 लीटर जूस है। इसमें से 5 लीटर जूस निकाल कर उसकी जगह 5 लीटर पानी डाल दिया जाता है। यह क्रिया और चार बार दोहराई जाती है। अंतिम प्रतिस्थापना के बाद बर्तन में कितना जूस बचा है ? $[(0{\cdot}9)^5 = 0{\cdot}59049$ लीजिए]

अथवा

1000 मीटर की एक दौड़ में A, B तथा C को क्रमश: सोना, चांदी तथा ब्रोंज के मैडल मिलते हैं। यदि A, B को 100 मीटर से हराता है तथा B, C को 100 मीटर से हराता है, तो A ने C को कितने मीटर से हराया ?

29. एक अनभिनत सिक्का 9 बार उछाला गया। निम्न के प्राप्त करने की प्रायिकता ज्ञात कीजिए :

(i) पूरे पाँच बार पट आना

(ii) कम से कम पाँच बार पट आना

(iii) अधिकतम पाँच बार पट आना

अथवा

माना एक यादृच्छया चुने गए दिन में एक व्यक्ति द्वारा टेलीविजन देखने में लगाए गए घंटों की सख्या X है। X के द्वारा प्राप्त मानों x_i की प्रायिकताएँ निम्न रूप में हैं, जहाँ k एक अज्ञात अचर है।

$$P(X = x_i) = \begin{cases} 0{\cdot}2, & \text{यदि } x_i = 0 \\ kx_i, & \text{यदि } x_i = 1 \text{ अथवा } 2 \\ k(5 - x_i), & \text{यदि } x_i = 3 \\ 0, & \text{अन्यथा} \end{cases}$$

(i) k का मान ज्ञात कीजिए।

(ii) $P(X = 2)$, $P(X \geq 2)$ तथा $P(X \leq 2)$ ज्ञात कीजिए।

27. A person can row a boat at 5 km/h in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing.

28. A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated four more times. What is the amount of juice left in the container after final replacement ? [Take $(0{\cdot}9)^5 = 0{\cdot}59049$]

OR

In a 1000-metre race, A, B and C get Gold, Silver and Bronze medals respectively. If A beats B by 100 metres and B beats C by 100 metres, then by how many metres does A beat C ?

29. A fair coin is tossed 9 times. Find the probability of getting.

(i) exactly 5 tails;

(ii) at least 5 tails;

(iii) at most 5 tails.

OR

Let X denote the number of hours a person watches T.V. during a randomly selected day. The probability that X can take the values x_i, has the following form, where k is some unknown constant.

$$P(X = x_i) = \begin{cases} 0{\cdot}2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k.

(ii) Find : $P(X = 2)$, $P(X \geq 2)$ and $P(X \leq 2)$.

30. एक परीक्षा में 2000 विद्यार्थियों ने भाग लिया। उनके द्वारा प्राप्त किए गए अंकों का बंटन प्रसामान्य (normal) है जिसका माध्य 30 तथा मानक विचलन 6·25 है। कितने विद्यार्थियों का निम्न अंक लेने का अनुमान है :

(i) 20 और 40 के बीच

(ii) 25 से कम

[$P(0 \le z \le 1{\cdot}6) = 0{\cdot}4452$, $P(0 \le z \le 0{\cdot}8) = 0{\cdot}2881$ लीजिए]

31. किसी शहर के एक क्षेत्र के 18 विद्यार्थियों के एक समूह के IQ's का माध्य 95 पाया गया, तथा मानक विचलन 10 पाया गया, जबकि शहर के एक अन्य क्षेत्र के 12 विद्यार्थियों के समूह के IQ's का माध्य 100 पाया गया तथा जिनका मानक विचलन 8 था। टैस्ट कीजिए कि क्या इन दो विद्यार्थियों के समूहों के IQ's में, 1% सार्थकता के स्तर पर, सार्थक अंतर है। [$t_{28}(0{\cdot}01) = 2{\cdot}763$ लीजिए]

खण्ड-घ

इस खंड में दीर्घ उत्तर (LA) प्रकार के प्रश्न हैं। प्रत्येक प्रश्न के 5 अंक हैं।

32. (a) ज्ञात कीजिए कि x के किन मानों के लिए, फलन $f(x) = x^4 - \frac{x^3}{3}$ निरंतर वर्धमान या निरंतर ह्रासमान है।

अथवा

(b) एक फर्म के कुल लागत फलन तथा माँग फलन

$$C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50 \text{ तथा } x = 100 - p$$

द्वारा प्रदत्त हैं।

(i) x के पदों में कुल राजस्व फलन ज्ञात कीजिए।

(ii) x के पदों में कुल लाभ फलन P ज्ञात कीजिए।

(iii) अधिकतम लाभ वाला x का मान ज्ञात कीजिए।

(iv) रुपए को मुद्रा की इकाई लेकर अधिकतम लाभ क्या है ?

30. 2000 students appeared in an examination. Distribution of marks is assumed to be normal with mean 30 and standard deviation 6·25. How many students are expected to get marks

(i) between 20 and 40 ?

(ii) less than 25 ?

[Use : $P(0 \leq z \leq 1{\cdot}6) = 0{\cdot}4452$, $P(0 \leq z \leq 0{\cdot}8) = 0{\cdot}2881$]

31. The mean of IQs of a group of 18 students from one area of a city was found to be 95 with a standard deviation of 10, while the mean of IQs of a group of 12 students from another area of the city was found to be 100 with a standard deviation of 8. Test whether there is a significant difference between the IQs of two groups of students at 1% level of significance. [Use : $t_{28}(0{\cdot}01) = 2{\cdot}763$]

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Determine for what values of x, the function $f(x) = x^4 - \frac{x^3}{3}$ is strictly increasing or strictly decreasing.

OR

(b) A firm has the following total cost and demand functions :

$C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50$ and $x = 100 - p$

(i) Find the total revenue function in terms of x.

(ii) Find the total profit function P in terms of x.

(iii) Find the profit maximizing level of output of x.

(iv) What is the maximum profit, taking rupee as a unit of money ?

33. (a) एक कंपनी ने ₹ 1,00,000 के ऋण के लिए ऋणशोधन निधि (sinking fund) का निर्माण किया जो 4 वर्ष में पूरा होता है। इस निधि के लिए योगदान वर्ष के अन्त में करना होता है। प्रत्येक वार्षिक योगदान की राशि ज्ञात कीजिए जबकि ब्याज की दर 18% वार्षिक है। $[(1{\cdot}18)^4 = 1{\cdot}9388$ लीजिए]

अथवा

(b) एक फर्म ने एक मशीनरी ₹ 7,40,000 में 1 अप्रैल, 2020 को खरीदी तथा इसके लगाने पर ₹ 60,000 खर्च किए। इसकी लाभप्रद आयु 5 वर्ष अनुमानित की गई है। 5 वर्ष के अन्त में इसका अवशिष्ट मूल्य ₹ 40,000 अनुमानित है। वार्षिक अवमूल्यन (मूल्यह्रास) राशि तथा अवमूल्यन (मूल्यह्रास) की दर ज्ञात कीजिए।

34. एक व्यक्ति ₹ 10,00,000 का गृह ऋण 6% वार्षिक दर पर लेता है जबकि ब्याज मासिक संयोजन होता है। उसने उस ऋण को 15 वर्षों में समान मासिक किश्तों में चुकाने का निर्णय लिया। उसकी ई.एम.आई. (EMI) निम्न विधि से ज्ञात कीजिए :

(i) समान दर विधि से,

(ii) घटते बकाया विधि से।

[दिया है : $(1{\cdot}005)^{-180} = 0{\cdot}4074824$]

35. एक लाइब्रेरी को एक शेल्फ़ पर दो भिन्न प्रकार की पुस्तकों को रखना है। यह पुस्तकें प्रत्येक क्रमश: 6 cm तथा 4 cm मोटी हैं तथा प्रत्येक का भार क्रमश: 1 kg तथा $1\frac{1}{2}$ kg है। शेल्फ़ 96 cm लंबा है तथा अधिकतम 21 kg भार सहन कर सकता है। शेल्फ़ पर अधिकतम पुस्तकें आ जाएँ उसके लिए दोनों प्रकार की पुस्तकों को शेल्फ़ पर किस प्रकार रखा जाए ? उसे एक रैखिक प्रोग्रामन समस्या के रूप में लिखिए तथा आलेख द्वारा हल कीजिए।

33. (a) A company establishes a sinking fund to provide for the payment of ₹ 1,00,000 debt, maturing in 4 years. Contributions to the fund are to be made at the end of year. Find the amount of each annual deposit if interest is 18% per annum. [Use $(1{\cdot}18)^4 = 1{\cdot}9388$]

OR

(b) A firm bought a machinery for ₹ 7,40,000 on 1st April, 2020 and ₹ 60,000 is spent on its installation. Its useful life is estimated to be of 5 years. Its scrap value at the end of 5 years is estimated to be ₹ 40,000. Find the amount of annual depreciation and the rate of depreciation.

34. A person takes a housing loan worth ₹ 10,00,000 at an interest rate of 6% p.a compounded monthly. He decided to repay the loan by equal monthly instalments in 15 years. Calculate the EMI, using

(i) flat rate method,

(ii) reducing balance method.

[Given : $(1{\cdot}005)^{-180} = 0{\cdot}4074824$]

35. A library has to accommodate two different types of books on a shelf. The books are each 6 cm and 4 cm thick and each weighs 1 kg and $1\frac{1}{2}$ kg respectively. The shelf is 96 cm long and can support a weight of atmost 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books ? Formulate it as an L.P.P and so solve it graphically.

खण्ड ई

इस खंड में 3 प्रश्न हैं जो कि प्रकरण अध्ययन आधारित हैं। प्रत्येक प्रश्न के 4 अंक हैं।

प्रकरण अध्ययन–1

36. एक विद्यालय से नैतिक मूल्यों के आधार पर पुरस्कार देने के लिए 10 विद्यार्थियों को चुना गया तथा उन्हे तीन वर्गों में बाँटा गया। पहले वर्ग में सख्त मेहनत करने वालों को रखा गया, दूसरे वर्ग में ईमानदार तथा कानून को मानने वालों को रखा गया तथा तीसरे वर्ग में सतर्क तथा आज्ञाकारी विद्यार्थियों को रखा गया। पहले वर्ग के विद्यार्थियों की संख्या के दुगुने को दूसरे वर्ग के विद्यार्थियों की संख्या में जोड़ने पर 13 प्राप्त होता है जबकि पहले तथा दूसरे वर्गों की कुल संख्या, तीसरे वर्ग की संख्या के चार गुने के बराबर है। पहले, दूसरे तथा तीसरे वर्ग में विद्यार्थियों की संख्या क्रमश: x, y तथा z ली गई हैं।

उपरोक्त के आधार पर निम्न के उत्तर दीजिए :

(a) उपरोक्त से लिखे जा सकने वाले रैखिक समीकरण निकाय को लिखिए। *1*

(b) गुणांकों का आव्यूह (A) लिखिए । *1*

(c) (i) आव्यूह A के अवयवों के सहखंडों का आव्यूह लिखिए। *2*

अथवा

(c) (ii) प्रत्येक वर्ग में विद्यार्थियों की संख्या ज्ञात कीजिए। *2*

प्रकरण अध्ययन–2

37. एक कंपनी यह नोट करती है कि किसी विशेष उत्पाद जो यह कम्पनी बनाती है की बिक्री अधिकतम होती है जब उसका लिए जाने वाला मूल्य कम कर दिया जाए। परिणामस्वरुप, बिक्री से प्राप्त राजस्व शुरू में बढ़ता है जब बिकने वाले उत्पाद की संख्या बढ़ती है, यह एक उच्चतम बिंदु पर पहुँचता है तथा फिर गिरता है। राजस्व का पैटर्न $y = 40,00,000 - (x - 2000)^2$ द्वारा दर्शाया जा सकता है, जहाँ y कुल राजस्व है तथा x बिकने वाले उत्पादन की इकाइयाँ दर्शाता है।

उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

(a) ज्ञात कीजिए कि उत्पाद की कितनी इकाइयाँ बेचने पर अधिकतम राजस्व प्राप्त होता है। *2*

(b) इस अधिकतम राजस्व की राशि कितनी है ? *1*

(c) 2500 इकाइयाँ बेचने पर कितना राजस्व प्राप्त होगा ? *1*

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students, and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of the first and the second group is four times that of the third group. Assume that x, y and z denote the number of students in first, second and third group respectively.

Based on the above information, answer the following questions :

(a) Write the system of linear equations that can be formulated from the above described situation. *1*

(b) Write the coefficient matrix, say A. *1*

(c) (i) Write the matrix of cofactors of every element of matrix A. *2*

OR

(c) (ii) Determine the number of students of each group. *2*

Case Study – 2

37. A company notes that higher sales of a particular item, which it produced, is achieved by lowering the price charged. As a result, the total revenue from the sales at first rises as the number of units sold increases, reaches the highest point, and then falls off. The pattern of revenue is described by the relation $y = 40,00,000 - (x - 2000)^2$, where y is the total revenue and x is the number of units sold.

Based on the above information, answer the following questions :

(a) Find what number of units sold maximizes total revenue. *2*

(b) What is the amount of this maximum revenue ? *1*

(c) What would be the total revenue if 2500 units were sold ? *1*

प्रकरण अध्ययन–3

38. निम्न आँकड़े, उन ग्रामीण, शहरी तथा उप-शहरी भारतीयों की प्रतिशतता दर्शाते हैं जिनके पास घर पर उच्च स्पीड का इंटरनेट का कनेक्शन है।

वर्ष	ग्रामीण	शहरी	उप-शहरी
2016	3	9	9
2017	6	18	17
2018	9	21	23
2019	16	29	29
2020	24	38	40

उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

(a) ग्रामीण विद्यार्थियों के लिए, न्यूनतम वर्गों वाली विधि से सरल-रेखा ट्रेंड ज्ञात कीजिए। *2*

अथवा

शहरी भारतीयों के लिए, न्यूनतम वर्गों वाली विधि से सरल-रेखा ट्रेंड ज्ञात कीजिए। *2*

(b) ट्रेंड समीकरण के प्रयोग से वर्ष 2021 के लिए शहरी वर्ग के लिए भविष्यवाणी क्या है ? *1*

(c) ट्रेंड समीकरण के प्रयोग से वर्ष 2021 के लिए ग्रामीण वर्ग के लिए भविष्यवाणी क्या है ? *1*

Case Study – 3

38. The following data shows the percentage of rural, urban and sub-urban Indians who have high speed internet connection at home.

Year	Rural	Urban	Sub-urban
2016	3	9	9
2017	6	18	17
2018	9	21	23
2019	16	29	29
2020	24	38	40

Based on the above information, answer the following questions :

(a) Derive straight-line trend by the method of least squares for the rural students. *2*

OR

(a) Derive straight-line trend by the method of least squares for the urban Indians. *2*

(b) What is the forecast for the year 2021 for urban group using trend equation ? *1*

(c) What is the forecast for the year 2021 for rural group using trend equation ? *1*

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary School Supplementary Examination, July- **2023**
APPLIED MATHEMATICS PAPER CODE 465

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking scheme carries only suggested value points for the answers. These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right **(✓)** while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**
10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**

11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code–241)
(PAPER CODE: 465)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	If $100 \equiv x \pmod 7$, then the least positive value of x is : (a) 6 (b) 4 (c) 3 (d) 2	
Ans.	(d) 2	**1**
2.	In a kilometre race, A beats B by 50 metres or 10 seconds. The time taken by A to complete the race is : (a) 90 seconds (b) 120 seconds (c) 190 seconds (d) 200 seconds	
Ans.	(c) 190 seconds	**1**
3.	If a man rows 32 km downstream and 14 km upstream in 6 hours each, then the speed of the stream is : (a) 2 km/h (b) 1·5 km/h (c) 2·5 km/h (d) 2·25 km/h	
Ans.	(b) 1.5 km/h	**1**
4.	If $-3x + 17 < -13$, then : (a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$ (c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10]$	

Ans.	(a) $x \in (10, \infty)$	**1**
5.	If A and B are two matrices such that AB = A and BA = B, then B^2 is equal to : (a) B (b) A (c) I (d) O	
Ans.	(a) B	**1**
6.	If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ is a symmetric matrix, then : (a) $x = 0, y = 5$ (b) $x = 5, y = 0$ (c) $x = y$ (d) $x + y = 0$	
Ans.	(c) $x = y$	**1**
7.	The value of x for which the points $(2, -1)$, $(-3, 4)$ and $(x, 5)$ are collinear, is : (a) -4 (b) 4 (c) 2 (d) -2	
Ans.	(a) -4	**1**
8.	If $x + y = 8$, then the maximum value of xy is : (a) 12 (b) 16 (c) 20 (d) 24	
Ans.	(b) 16	**1**

9.	The function $f(x) = x^x$, $x > 0$ is decreasing in the interval : (a) $(-\infty, e)$ (b) $(0, e)$ (c) $\left(0, \frac{1}{e}\right)$ (d) $\left[\frac{1}{e}, \infty\right)$	
Ans.	(c) $(\mathbf{0}, \frac{\mathbf{1}}{e})$	**1**
10.	$\int \frac{1}{x + x\log x}\,dx$ is equal to : (a) $1 + \log x + C$ (b) $x + \log x + C$ (c) $x \log(1 + \log x) + C$ (d) $\log(1 + \log x) + C$	
Ans.	(d) $\log(1 + \log x) + C$	**1**
11.	If the supply function is $p = 4 + x$, then the producer's surplus when 12 units are sold, is : (a) 72 (b) 64 (c) 76 (d) 46	
Ans.	(a) 72	**1**
12.	The order of the differential equation corresponding to family of curves $y = Ae^{3x} + Be^{-3x}$ is : (a) 1 (b) 2 (c) 3 (d) 4	
Ans.	(b) 2	**1**
13.	If 'm' is the mean of Poisson distribution, then $P(r = 0)$ is given by : (a) e^{-m} (b) e^{m} (c) e (d) m^{-e}	

Ans.	(a) e^{-m}	**1**
14.	Normal distribution is symmetric about : (a) variance (b) standard deviation (c) mean (d) covariance	
Ans.	(c) mean	**1**
15.	For a student's t-test, the test statistic t is given by : (a) $t = \bar{x} - \mu$ (b) $t = \frac{\bar{x}}{s}$ (c) $t = \frac{\bar{x} - \mu}{s}$ (d) $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$	
Ans.	(d) $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$	**1**
16.	For the given values 15, 23, 28, 36, 41, 46, the 3-yearly moving averages are : (a) 24, 29, 35, 41 (b) 22, 28, 35, 41 (c) 22, 29, 35, 41 (d) 24, 28, 35, 41	
Ans.	(c) 22, 29, 35, 41	**1**
17.	The effective rate of return which is equivalent to nominal rate of 8% p.a. compounded quarterly is : [Given $(1{\cdot}02)^4 = 1{\cdot}0824$] (a) 8·16% (b) 7·95% (c) 8·24% (d) 8·5%	
Ans.	(c) 8.24 %	**1**

18.	The maximum value of the function $z = 7x + 5y$, subject to constraints $x \le 3, y \le 2, x \ge 0, y \ge 0$, is : (a) 21 (b) 10 (c) 31 (d) 37	
Ans.	(c) 31	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true but Reason (R) is false. (d) Assertion (A) is false but Reason (R) is true.	
19.	*Assertion (A) :* The slope of the normal to the curve $y = 2x^2 - 5x$ at $x = -1$ is -1. *Reason (R) :* The slope of the normal to the curve $y = f(x)$ at point (α, β) is given by $(x - \alpha) + \left(\frac{dy}{dx}\right)_{(\alpha, \beta)} \cdot (y - \beta) = 0$.	
Ans.	Both (A) and (R) are false, so 1 mark may be given to all who attempted	**1**
20.	*Assertion (A) :* If A is a square matrix of order 3 such that $\lvert \text{adj } A \rvert = 144$, then the value of $\lvert A \rvert$ is ± 12. *Reason (R) :* If A is an invertible matrix of order n, then $\lvert \text{adj } A \rvert = \lvert A \rvert^{n-1}$.	
Ans.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21(a).	Solve the inequality : $\frac{3}{5}x - \frac{2x-1}{3} > 1, x \in W$	

Ans.	$\frac{3}{5}x - \frac{2x-1}{3} > 1$ $\Rightarrow 9x - 5(2x-1) > 15$ Solving we get, $x < -10$ Solution set is $\emptyset$	**1** ½ ½
	OR	
21(b).	Solve the inequality : $$-\frac{2}{3} < -\frac{x}{3} + 1 \leq \frac{2}{3}, x \in R$$	
Ans.	$-\frac{2}{3} < -\frac{x}{3} + 1 \leq \frac{2}{3}$ $\Rightarrow -\frac{5}{3} < -\frac{x}{3} \leq -\frac{1}{3}$ $\Rightarrow \frac{1}{3} \leq \frac{x}{3} < \frac{5}{3}$ $\Rightarrow 1 \leq x < 5$ Solution set is $[1, 5)$	**1** **1**
22.	Write the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.	
Ans.	Let $A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$ $P = \frac{A+A'}{2} = \begin{bmatrix} 7 & -2 & -2 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ and $Q = \frac{A-A'}{2} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ So, $P' = P \Rightarrow$ symmetric, $Q' = -Q \Rightarrow$ skew symmetric $A = \frac{A+A'}{2} + \frac{A-A'}{2} = \begin{bmatrix} 7 & -2 & -2 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	½ ½+½ ½
23(a).	It is given that 2% of screws manufactured by a company are defective. Using Poisson distribution, find the probability that a packet of 100 screws contains no defective screw. (Given : $e^{-2} = 0{\cdot}14$)	
Ans.	Let p be the probability that the screw is defective	

	Then $p = \frac{2}{100}$. Here $n = 100$ So, $m = np = 2$ Let X denote the number of defective screws in a packet of 100 screws. Then X follows the probability distribution as $P(X = r) = e^{-m}\frac{m^r}{r!}, r = 0,1,2 \ldots$ P(no defective screw) $= P(X = 0) = e^{-m}\frac{m^0}{0!} = e^{-2} = 0.14$	**1** **1**
	OR	
23(b).	If the standard deviation of a Poisson variable X is $\sqrt{3}$, then find P (X > 0). [Use : $e^{-3} = 0{\cdot}05$]	
Ans.	Let X follows Poisson distribution with parameter m Then Variance $= m = (\sqrt{3})^2 = 3$ $\therefore P(X = r) = e^{-m}\frac{m^r}{r!} = e^{-3}\frac{3^r}{r!}, r = 0,1,2 \ldots$ Hence $P(X > 0) = 1 - P(X = 0) = 1 - e^{-3} = 1 - 0.05 = 0.95$	½ **1½**
24.	Find the present value of a sequence of payments of ₹ 1000 made at the end of every 6 months and continuing forever, if money is worth 8% per annum compounded semi-annually.	
Ans.	The given annuity is a perpetuity of first type in which $R =$ ₹ 1000 and $r = \frac{8}{2}\% = 4\%$ per half year So, $i = \frac{4}{100} = 0.04$ Present value $= P = \frac{R}{i} = \frac{1000}{0.04} = 25000$ Hence the present value is ₹ 25000	**1** **1**
25.	The value of a machine purchased two years ago, depreciates at the annual rate of 10%. If its present value is ₹ 97,200, find. (i) its value after 3 years; (ii) its value when it was purchased.	
Ans.	Given $P =$ ₹ 97,200, $r = 10\%$ p.a $\Longrightarrow i = \frac{10}{100} = 0.1$ (i) Value after 3 years = Present value $\times (1 - 0.1)^3$	

	$= ₹\, 97200(0.9)^3$ $= ₹\, 70858.80$ (ii) Present value = value 2 years ago $\times (1-0.1)^2$ $\Rightarrow 97200 =$Value 2 years ago $\times (0.9)^2$ $\Rightarrow$ Value 2 years ago $= ₹\, \frac{97200}{(0.9)^2} = ₹\, 120000$	**1** **1**
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	
26.	Formulate the following problem as an LPP : A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit.	
Ans.	Let $\boldsymbol{x}$ and $\boldsymbol{y}$ denote the number of rings and chains respectively Maximize $\boldsymbol{Z = 300\,x + 190\,y}$ Subject to constraints $\boldsymbol{x + y \leq 24}$ $\boldsymbol{x + \frac{y}{2} \leq 16}$ $\boldsymbol{x, y \geq 0}$	**1** **2**
27.	A person can row a boat at 5 km/h in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing.	
Ans.	Let the rate at which the stream is flowing be x km/h and let the distance covered by the boat be y km According to question, $\frac{3y}{5+x} = \frac{y}{5-x}$ $\Rightarrow 3(5-x) = 5+x$ $\Rightarrow x = 2.5$ Thus, the stream is flowing at the rate of 2.5 km/h	**2** **1**

28(a).	A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated four more times. What is the amount of juice left in the container after final replacement ? [Take $(0{\cdot}9)^5 = 0{\cdot}59049$]	
Ans.	Total juice in the container = 50 litres Juice taken out = 5 litres No. of times process repeated = 5 Amount of juice in container after final replacement $= \mathbf{50}\left(\mathbf{1} - \frac{\mathbf{5}}{\mathbf{50}}\right)^{\mathbf{5}}$ $= \mathbf{29.52}$ litres	**2** **1**
28(b).	In a 1000-metre race, A, B and C get Gold, Silver and Bronze medals respectively. If A beats B by 100 metres and B beats C by 100 metres, then by how many metres does A beat C ?	
Ans.	A beats B by 100 metres, means A travels 1000 metres in the same time in which B travels 900 metres. B beats C by 100 metres, means B travels 1000 metres in the same time in which C travels 900 metres. $\therefore A : B = 10 : 9$ $B : C = 10 : 9$ $\Longrightarrow \mathrm{A} : \mathrm{B} : \mathrm{C} = 100 : 90 : 81$ So, A travels 100 metres and in the same time C travels 81 metres Thus, A beats C by 190 metres	**1** **1** **1**
29(a).	A fair coin is tossed 9 times. Find the probability of getting. (i) exactly 5 tails; (ii) at least 5 tails; (iii) at most 5 tails.	

Ans.

Repeated tosses of a fair coin qualify as Bernoulii's trials

Let X denote the number of trails in an experiment of such trials and hence is the binomial distribution

Here $n = 9, p = \frac{1}{2}$ and $q = 1 - \frac{1}{2} = \frac{1}{2}$

(a) P(exactly 5 success) $= P(X = 5) = 9_{C_5} p^5 q^4 = 9_{C_5} \left(\frac{1}{2}\right)^9$

$= \frac{63}{256}$ **1**

(b) P(at least 5 successes) $= P(X \geq 5)$

$= \left(\frac{1}{2}\right)^9 [9_{C_5} + 9_{C_6} + 9_{C_7} + 9_{C_8} + 9_{C_9}]$

$= \frac{256}{512} = \frac{1}{2}$ **1**

(c) P(at most 5 successes) $= P(X \leq 5) = 1 - P(X > 5)$

$= 1 - \left(\frac{1}{2}\right)^9 [9_{C_6} + 9_{C_7} + 9_{C_8} + 9_{C_9}]$

$= 1 - \frac{130}{512} = \frac{382}{512} = \frac{191}{256}$ **1**

OR

29(b).

Let X denote the number of hours a person watches T.V. during a randomly selected day. The probability that X can take the values x_i, has the following form, where k is some unknown constant.

$$P(X = x_i) = \begin{cases} 0{\cdot}2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k.

(ii) Find : P(X = 2), P(X ≥ 2) and P(X ≤ 2).

Ans.

x_i	0	1	2	3	otherwise
$P(x_i) = p_i$	0.2	k	$2k$	$2k$	0

½

(i) As $\sum p_i = 1$, we have

$0.2 + k + 2k + 2k = 1$

$\Rightarrow 5k = 0.8 \Rightarrow k = 0.16$ **1**

	(ii) $P(X = 2) = 2k = 0.32$ $P(X \geq 2) = 2k + 2k = 0.64$ $P(X \leq 2) = 0.2 + k + 2k = 0.68$	½ ½ ½
30.	2000 students appeared in an examination. Distribution of marks is assumed to be normal with mean 30 and standard deviation 6·25. How many students are expected to get marks (i) between 20 and 40 ? (ii) less than 25 ? [Use : P(0 ≤ z ≤ 1·6) = 0·4452, P(0 ≤ z ≤ 0·8) = 0·2881]	
Ans.	Let X denote the marks of the student. Mean $(\mu) = 30$, S.D $(\sigma) = 6.25$ $Z = \frac{X - \mu}{\sigma} = \frac{X - 30}{6.25}$ (i) When $X = 20, Z = -1.60$ When $X = 40, Z = 1.60$ $\therefore P(20 \leq X \leq 40) = P(-1.60 \leq Z \leq 1.60)$ $= P(-1.60 \leq Z \leq 0) + P(0 \leq Z \leq 1.60)$ $= 2P(0 \leq Z \leq 1.60)$ $= 2 \times 0.4452 = 0.8904$ Thus, out of 2000 students, the expected number of students getting marks between 20 and 40 $= 2000 \times 0.8904 = 1780.8$ or 1781 (ii) When $X = 25, Z = -0.80$ So, $P(X < 25) = P(Z < -0.8) = P(Z > 0.8)$ $= P(Z \geq 0) - P(0 \leq Z \leq 0.8)$ $= 0.5 - 0.2881 = 0.2119$ Thus, out of 2000 students the expected number of students getting marks less than 25 $= 2000 \times 0.2119 = 423.8$ or 424	½ ½ 1 1
31.	The mean of IQs of a group of 18 students from one area of a city was found to be 95 with a standard deviation of 10, while the mean of IQs of a group of 12 students from another area of the city was found to be 100 with a standard deviation of 8. Test whether there is a significant difference between the IQs of two groups of students at 1% level of significance. [Use : $t_{28}(0{\cdot}01) = 2{\cdot}763$]	
Ans.	Here $n_1 = 18, \bar{x} = 95, s_1 = 10$ and $n_2 = 12, \bar{y} = 100, s_2 = 8$	

	$\therefore s = \sqrt{\frac{n_1 {s_1}^2 + n_2 {s_2}^2}{n_1 + n_2 - 2}} = \sqrt{\frac{18 \times 10^2 + 12 \times 8^2}{18 + 12 - 2}}$ $= \sqrt{\frac{2568}{28}} = 9.58$ Let H_0= no significant difference between IQs of two group of students H_α= significant difference between IQs of two group of students The test statistic $'t'$ is $t = \frac{\bar{x} - \bar{y}}{s} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$ $= \frac{95 - 100}{9.58} \times \sqrt{\frac{18 \times 12}{18 + 12}} = -\frac{5}{9.58} \times 2.68 = -1.398$ $\Rightarrow \lvert t \rvert = 1.398$ The test statistics t follows student's t-distribution with $n = 18 + 12 - 2$ i.e., 28 degrees of freedom At 1% level of significance, we have $t_{28}(0.01) = 2.763$ As $1.398 < 2.763$, null hypothesis is accepted Hence, there is no significance difference between IQs of two groups	1½ 1 ½
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32(a).	Determine for what values of x, the function $f(x) = x^4 - \frac{x^3}{3}$ is strictly increasing or strictly decreasing.	
Ans.	$f(x) = x^4 - \frac{x^3}{3} \Rightarrow f'(x) = 4x^3 - x^2 = x^2(4x - 1)$ $f'(x) = 0 \Rightarrow x = 0, \frac{1}{4}$ Thus, critical points divide R into three parts $x < 0, 0 < x < \frac{1}{4}, x > \frac{1}{4}$ When $x < 0, f'(x)$ is $-$ve $\therefore f(x)$ is strictly decreasing for $x < 0$ When $0 < x < \frac{1}{4}, f'(x)$ is $+$ve $\therefore f(x)$ is strictly increasing for $x < 0$ When $x > \frac{1}{4}, f'(x)$ is $-$ve $\therefore f(x)$ is strictly decreasing for $x < 0$ Hence $f(x)$ is strictly increasing on $\left(\frac{1}{4}, \infty\right)$ and strictly decreasing on $(-\infty, 0) \cup \left(0, \frac{1}{4}\right)$	1 ½ 1 1 1 ½

	OR	
32(b).	A firm has the following total cost and demand functions : $C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50$ and $x = 100 - p$ (i) Find the total revenue function in terms of x. (ii) Find the total profit function P in terms of x. (iii) Find the profit maximizing level of output of x. (iv) What is the maximum profit, taking rupee as a unit of money ?	
Ans.	Here $\boldsymbol{C = \frac{x^3}{3} - 7x^2 + 111x + 50}$ and $\boldsymbol{x = 100 - p}$ i.e., $\boldsymbol{p = 100 - x}$ (i) R, the revenue function is $\boldsymbol{R = px = (100 - x)x = 100\,x - x^2}$ (ii) Profit function $\boldsymbol{P(x) = R(x) - C(x)}$ $\boldsymbol{= (100\,x - x^2) - \left(\frac{x^3}{3} - 7x^2 + 111x + 50\right)}$ $\boldsymbol{= -\frac{x^3}{3} + 6x^2 - 11x - 50}$ (iii) $\boldsymbol{\frac{dP}{dx} = -x^2 + 12x - 11}$ For $\boldsymbol{P}$ to be maximum, $\boldsymbol{\frac{dP}{dx} = 0 \Rightarrow x = 1, 11}$ $\boldsymbol{\frac{d^2P}{dx^2} = -2x + 12 > 0}$ at $\boldsymbol{x = 1}$ and < 0 at $\boldsymbol{x = 11}$ Thus $\boldsymbol{P}$ is maximum when $\boldsymbol{x = 11}$ Hence, the profit maximising level of output is 11 units (iv) Maximum profit $= [\boldsymbol{P(x)}]_{x=11}$ $\boldsymbol{= -\frac{(11)^3}{3} + 6(11)^2 - 11(11) - 50}$ $\boldsymbol{= 111.33\ or\ 111}$	½ 1 ½ 1 1 1
33(a).	A company establishes a sinking fund to provide for the payment of ₹ 1,00,000 debt, maturing in 4 years. Contributions to the fund are to be made at the end of year. Find the amount of each annual deposit if interest is 18% per annum. [Use $(1{\cdot}18)^4 = 1{\cdot}9388$]	
Ans.	Let each annual deposit to the sinking fund be ₹ R $\therefore\ \boldsymbol{100000 = R\left[\frac{(1+0.18)^4 - 1}{0.18}\right]}$ $\boldsymbol{= R\left[\frac{(1.18)^4 - 1}{0.18}\right]}$	2

	$= R\left[\frac{0.4388}{0.18}\right] = R(5.2186)$ $\Longrightarrow \mathbf{R} = \frac{100000}{5.2186} = ₹\,19162.22$	1½ 1½
	OR	
33(b).	A firm bought a machinery for ₹ 7,40,000 on 1st April, 2020 and ₹ 60,000 is spent on its installation. Its useful life is estimated to be of 5 years. Its scrap value at the end of 5 years is estimated to be ₹ 40,000. Find the amount of annual depreciation and the rate of depreciation.	
Ans.	$C = ₹\,7,40,000 + ₹\,60,000 = ₹\,8,00,000$ And $S = ₹\,40,000$ $\therefore$ Annual depreciation $= \frac{C-S}{n} = ₹\,\frac{7,60,000}{5} = ₹\,1,52,000$ Rate of depreciation $= \frac{D}{C-S} \times 100$ $= \frac{1,52,000}{8,00,000-40,000} \times 100$ $= 20\%$	1 1 1 1 1
34.	A person takes a housing loan worth ₹ 10,00,000 at an interest rate of 6% p.a compounded monthly. He decided to repay the loan by equal monthly instalments in 15 years. Calculate the EMI, using (i) flat rate method, (ii) reducing balance method. [Given : $(1{\cdot}005)^{-180} = 0{\cdot}4074824$]	
Ans.	Here, $P = ₹\,10,00,000, i = \frac{6}{12\times100} = 0.005$ $n = 15$ years $= 180$ months (i) Using flat rate method EMI $= P\left(i + \frac{1}{n}\right)$ $= 10,00,000\left(0.005 + \frac{1}{180}\right) = ₹\,10555.55$ (ii) Using reducing balancing method EMI $= \frac{Pi}{1-(1+i)^{-n}}$ $= \frac{10,00,000\times0.005}{1-(1.005)^{-180}}$	1 1+½ 1

	$= \frac{5000}{1-0.4704824} = ₹\,8438.57$	**1½**
35.	A library has to accommodate two different types of books on a shelf. The books are each 6 cm and 4 cm thick and each weighs 1 kg and $1\frac{1}{2}$ kg respectively. The shelf is 96 cm long and can support a weight of atmost 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books ? Formulate it as an L.P.P and so solve it graphically.	
Ans.		

Types of boxes	Thickness (in cm)	Weight (in kg)
Type 1	6	1
Type 2	4	$1\frac{1}{2}$
Max Availability	96 cm	21 kg

Let the two types of boxes be x and y respectively

Let Z denote the maximum number of books that can be accommodated in the shelf

LPP is

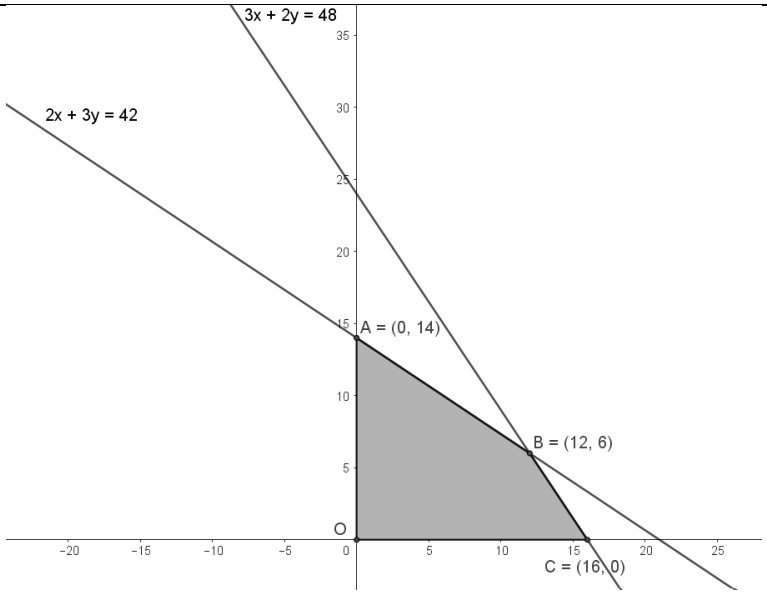
$\therefore Z = x + y$ **1**

Subject to constraints

$6x + 4y \leq 96$ or $3x + 2y \leq 48$

$x + \frac{3}{2}y \leq 21$ or $2x + 3y \leq 42$ **1½**

$x, y \geq 0$

	 Here, $(Z)_A = 14, (Z)_B = 18, (Z)_C = 16$ So, Z is maximum at B Hence, the shelf should be filled with 12 books of type 1 and 6 books of type 2	**1½ For graph with correct region** **1**
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	

36. **Case Study – 1**

10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students, and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of the first and the second group is four times that of the third group. Assume that x, y and z denote the number of students in first, second and third group respectively.

Based on the above information, answer the following questions :

(a) Write the system of linear equations that can be formulated from the above described situation. *1*

(b) Write the coefficient matrix, say A. *1*

(c) (i) Write the matrix of cofactors of every element of matrix A. *2*

OR

(c) (ii) Determine the number of students of each group. *2*

Ans.

(a) $x + y + z = 10, 2x + y = 13, x + y = 4z$ **1**

(b) coefficient matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}$ **1**

(c) (i) Cofactor matrix of $A = \begin{bmatrix} -4 & 8 & 1 \\ 5 & -5 & 0 \\ -1 & 2 & -1 \end{bmatrix}$ **2**

OR

(ii) Adj (A) $= \begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$ and $|A| = 5 \neq 0$

$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$ **1**

Thus, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 10 \\ 13 \\ 0 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 25 \\ 15 \\ 10 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$

$\therefore$ 5 students in first group, 3 students in second group and 2 students in third group **1**

37.	**Case Study – 2** A company notes that higher sales of a particular item, which it produced, is achieved by lowering the price charged. As a result, the total revenue from the sales at first rises as the number of units sold increases, reaches the highest point, and then falls off. The pattern of revenue is described by the relation $y = 40,00,000 - (x - 2000)^2$, where y is the total revenue and x is the number of units sold. Based on the above information, answer the following questions : (a) Find what number of units sold maximizes total revenue. *2* (b) What is the amount of this maximum revenue ? *1* (c) What would be the total revenue if 2500 units were sold ? *1*	
Ans.	(a) $y = 40,00,000 - (x - 2000)^2$ Gives $\frac{dy}{dx} = -2(x - 2000)$ So, $\frac{dy}{dx} = 0$ when $x = 2000$ $\frac{d^2y}{dx^2} = -2 < 0$ Hence y is max at $x = 2000$ (b) Max Revenue $= 40,00,000 - (2000 - 2000)^2 = 40,00,000$ (c) Total Revenue $= 40,00,000 - (2500 - 2000)^2$ $= 37,50,000$	**1½** **½** **1** **1**

38.

Case Study – 3

The following data shows the percentage of rural, urban and sub-urban Indians who have high speed internet connection at home.

Year	Rural	Urban	Sub-urban
2016	3	9	9
2017	6	18	17
2018	9	21	23
2019	16	29	29
2020	24	38	40

Based on the above information, answer the following questions :

(a) Derive straight-line trend by the method of least squares for the rural students. *2*

OR

(a) Derive straight-line trend by the method of least squares for the urban Indians. *2*

(b) What is the forecast for the year 2021 for urban group using trend equation ? *1*

(c) What is the forecast for the year 2021 for rural group using trend equation ? *1*

Ans.

(a)

y	x	x^2	xy
3	-2	4	-6
6	-1	1	-6
9	0	0	0
16	1	1	16
24	2	4	48
$\sum y = 58$		$\sum x^2 = 10$	$\sum xy = 52$

1½

Trend value is $y = \frac{\sum y}{5} + \frac{\sum xy}{\sum x^2} x$

$y = 11.6 + 5.2\,x$ ½

OR

y	x	x^2	xy
9	-2	4	-18
18	-1	1	-18
21	0	0	0
29	1	1	29
38	2	4	76
$\sum y = 115$		$\sum x^2 = 10$	$\sum xy = 69$

1½

Trend value is $y = \frac{115}{5} + \frac{69}{10}x$ ½

$y = 23 + 6.9\,x$

(b) For $x = 3$, we have

$y = 23 + 6.9\,(3) = 43.7$ 1

(c) For $x = 3$, we have

$y = 11.6 + 5.2\,(3) = 27.2$ 1